STUDIES AND RESEARCHES ON EROSION OF THE BAD RIVER

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Abstract

The paper presents a way of analyzing the erosion phenomenon of riverbeds. Theoretical study and experimental research was carried out on a sector of the Moldova River in the area of Sochi, Iasi County. The study of the stability of the river bed was carried out by analyzing the value of the shear stress on the perimeter watered in characteristic cross sections. The river bank has been measured topographically in transverse and longitudinal profiles positioned on the river section (at the entrance, exit, and imposed sections). For analysis of bed stability was determined the wet perimeter distribution of the hydraulic parameters (v, h and n) in the imposed sections. For the study of the erosion of the bed, a theoretical hydraulic-mathematical model was developed. The analysis model resolves in the first stage the equation of the progressive variable fluctuations in riverbeds by the finite difference method. In the second stage, shear stresses are determined at characteristic points on the perimeter of the bed. The hydraulic - mathematical model (coded MPGV_ River_Bed.m) was solved in the MATLAB programming environment. The obtained results allowed the erosion zones to be highlighted in the river bed.

Keywords: shear effort, computational model, cohesive rocks, stability

The Romanian rivers have in recent years a hydrological regime with a very large variation in flows. Floods have produced important morphological changes in the minor bed and have affected the stability of the bed constructions. The study of morphology of riverbeds is of particular importance for the agricultural sector in view of the location of arable land in the riverine area. The morphological modification of the river bed influences the location of the water catchments for irrigation, pumping stations, dikes, pumping stations for drainage, roads, etc. Morphological risk areas in the bed and river banks can also be studied by analyzing the erosion phenomenon produced by the flow of water at normal flows and flood flows.

The prognosis of hydrodynamic erosions provides data on the evolution of the minor and major river beds under the influence of hydrological factors (Thorne C., 1982). For the studied section of the river, there are drawn out plans, cross-sections and longitudinal profiles, forecasting plans for the modification of riparian lands, etc. The resulting data is used to predict the evolution of arable land that may be affected by the erosion phenomenon. The study data obtained serves to design sites for the economic and social objectives of the river's riparian area. Also, the data obtained is used in the design of shore defence works and riverside regulation works (Ferguson R. F., 2007).

The study of hydrodynamic erosion phenomena is particularly useful for rivers formed in layers of poor cohesive rock. Layers of this type consist of sludge cones, ballast and gravel layers, alluvial deposits, etc. Models of hydrodynamic erosion analysis are of the classical type (Maik 11, Hec-Ras, Mohid, etc.), or can be designed in the Mathlab programming language.

MATERIAL AND METHOD

The studies and researches were carried out on a section of the Moldova river located in the area of Sochi, Iaşi County (*figure1*). River bedding works and shoreline constructions have been executed on the river section. Hydrotechnical works were executed to protect the sub-crossing construction of the water supply pipes that feed water in Iasi city. The shore defence was made of concrete slabs supported on a large stone prism. The land in the coastal area is occupied by grassland on the left bank and partly by pasture and agricultural crops on the right bank.

The river section presents an island in the study area that changes the layout of the bed. Albia presents a unique route in the beginning, after which it branches into two arms and then returns to a single bed. The left arm of the river is more developed than the right arm (*figure 2*). The length of the bed in the study is 420 m, of which the island has a length of 274 m. The width of the minor bed ranges from 260 m upstream, 127 m on

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the left arm, 74 m on the right arm and 141 downstream. The depth of the bed ranges from 4.74 m to 4.32 m. The depth of the bed varies in different ways on the two arms (the right arm has the highest depths) (*figure 3*). Flow in the river is characterized by frequent temporal and spatial discontinuities of hydrological parameters (Luca M., 2012).

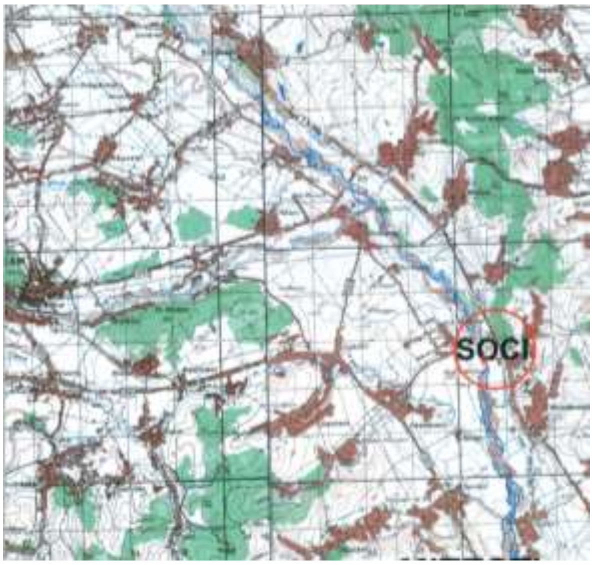

Figure 1 **The location of the research area on the Moldova River**

The Moldavian River presents in the study area a barely formed ballast massif with a thickness of 15 ... 20 m. The ballast layer extends into the main bed over distances of 200 ... 600 m. The non-cohesive material of the bed enables the formation of an active hydrodynamic erosion process (Luca M., Ignat A., 2007, Luca M., Stoenescu I., 2007).

Figure 2 **Images from the location study area on the Moldova river; view upstream of the left arm** (photo: Luca M., year 2012).

Research methodology consists of three stages of work. So (Luca M., 2012):

A. Stage 1 consists of field and laboratory studies to collect the data needed to run the hydrodynamic erosion simulation program. In the research area on the Moldova River, the following studies were carried out: topographic, hydrological, geotechnical, hydraulic.

B. Stage 2 consists of studies on hydrodynamic erosion on the analysis section of the Moldova River and the elaboration of the hydraulic - mathematical model for the analysis of the erosion phenomenon in river beds.

Figure 3 **Downstream view of the study on the Moldova River** (photo: Luca M., year 2012).

C. Stage 3 consists of the application of the hydrodynamic erosion analysis program on the study section of the Moldova River. The program is applied for a range of characteristic flows recorded on the study section.

The obtained results are compared with the data obtained by field measurements and those calculated.

RESULTS AND DISCUSSIONS

The modelling methodology requires the following steps: a) the conceptual model; b) the mathematical model; c) the numerical model; d) model calibration; e) model validation.

The analysis models describing the morphological processes that occur in the riverbed are characterized by the following:

- temporal coordination, indispensable for addressing dynamic/evolutionary processes;
- the spatial side, referring to the resolution of the geometrical meshing of the riverbed;
- the number of status variables / parameters that reflect the processes in the bed.

In the study of river bed stability, the size of shear stress on the wet perimeter is a determining factor. The shearing effort on the watered perimeter of the bed was assimilated with that at the bottom of the bed. For the shearing effort at the bottom of the bed was considered the expression (Cioc D., 1981):

$$\tau_b = g \cdot \rho \cdot \frac{n^2 \cdot v^2}{h^{1/3}} \tag{1}$$

where: τ_b is the shear stress (Pa); ρ - water density ($kg \cdot m^{-3}$); g - gravitational acceleration ($m \cdot s^{-2}$); n - Manning coefficient ($s \cdot m^{-1/3}$); v - velocity ($m \cdot s^{-1}$); h - depth of water (m).

The mean water velocity, given by Chezy's formula (Cioc D., 1981), in which the coefficient *C* was expressed by Manning's formula, and the hydraulic radius approximated with the depth *h*, has the following

(2) $$v = \frac{1}{n} h^{2/3} \sqrt{I_h}$$

where I_h is the hydraulic slope.

From the above equation, it follows:

(3) $$\frac{n^2 v^2}{h^{1/3}} = h I_h$$

Since g·ρ ≈ const and the product h Ih is an increasing function of flow, it follows from the relations (1) and (3) that the shear stress at the bottom τb is also an increasing flow function.

For the analysis of the stability of the bed, the distribution on the water perimeter of the bed of the hydraulic parameters v, h and n shall be determined. The analysis is performed for the flows characteristic of the hydrodynamic erosion phenomenon. To solve this problem a theoretical model of hydraulic-mathematical type has been developed (the model develops a model presented by Zaharia C., 2013).

In modelling, it was admitted that the movement is variable unevenly gradual, and the bed section is straight and the cross-section is stable. From a dimensional point of view, it was considered a pseudo-two-dimensional model. The cross section of the bed was schematized on work sectors. Average speeds were determined for each sector, depending on its average depth.

The geometric characteristics of the riverbed are reproduced by:

- the longitudinal profile drawn by the bedside,

(4) $$\left(x_j, z_{tv\,j}\right),\ j = 1,2,\cdots,N$$

- the cross-sections j in the points (6.3), represented by an open broken line, having M nodes, of coordinates:

(5) (y_i, z_i), i = 1,2,…*M*

On the basis of the coordinates (6.3), the slopes slope I_j can be defined on the slopes section delimited by the consecutive cross-sections *j* and *j + 1*,

(6) $$I_{j,j+1} = -\frac{z_{tv\,j+1} - z_{tv\,j}}{x_{j+1} - x_j},\ \text{cu}\ j = 1,2,\cdots,N-1$$

The hydraulic parameters of the river in the analysis section are: water level, *Z*, $Z \in (Z_{tv\,j}, Z_{n,\,Ind\,j})$, where $Z_{n,\,Ind\,j}$ is the non-inundability quota in section *j*; section (wet), *A*; Perimeter wet, *P*; hydraulic radius (mean), *Rh = A/P*; width of water mirror, B; average water depth, $\bar{h} = A/B$; relative width (water mirror) *βr*, $\beta_r = B/\bar{h} = B^2/A$; roughness coefficient *n* (after Manning, Forchheimer, Pavlovski, etc.); when rugosity is uneven, values of coefficient n must be specified in all nodes (5) by the set of values n_i, with i = 1,2,... M; speed module, *W*; flow module, *K*; current flow rate, *Q*; average speed, *V = Q/A*.

In any cross-section *j*, all hydraulic parameters are dependent on the water level *Z, Z*. The horizontal line *z = Z* intersects the broken line joining the nodes (5) as well as the upward-facing half-vertices passing through the nodes (5) with $z_i < Z$, in points M_Z,

(7) $M_Z = 2 \cdot M_Z^* = \nu_D - \nu_S + 1,\ M_Z \leq M$ °i $M_z^* \in n$

of coordinates:

(8) $(y_S = y_{vS},\ z_S = Z)$, $(y_i,\ Z)$ and $i \in [\nu_S + 1,\ \nu_D - 1]$, $(y_D = y_{vD},\ z_D = Z)$,

where n is the set of natural numbers, and ν_s and ν_D also are the order numbers of the nodes (4) on the right-hand, left-hand and right-bordered segments intersecting the line *z = Z*, whose quotas check the conditions:

(9) $z_{\nu_S} \geq Z,\ z_{\nu_S+1} < Z$ °i $z_{\nu_D} \geq Z,\ z_{\nu_D-1} < Z$

The depth of water in the nodes (4), which checks the condition, shows the following values:

(10) $$h_i = Z - z_i,\ \text{cu}\ i \in \left[\nu_S + 1, \nu_D - 1\right]$$

The points (8) and nodes (10) delineate M_Z - 1 trapezes (the triangles can be assimilated to trapezes having a small null base), for which the equations for the following sizes were deduced: mean depths, sectional areas and wet perimeters (Balan A., 2016).

The wetted section *A* and the wetted perimeter *P* are evaluated with the sums:

(11) $$A = \sum_{i=1}^{M_Z-1} A_i \ \text{şi}\ P = \sum_{i=1}^{M_Z-1} P_i$$

and the width *B* is given by the relation:

(12)
$$B = \left(y_{\nu_S+1} - y_S\right) + \sum_{2\cdot i^* \in [\nu_S+1, \nu_D-1]} \left(y_{2\cdot i^*} - y_{2\cdot i^*-1}\right) + \left(y_D - y_{\nu_D-1}\right)$$

The hydraulic rays of those M_Z -1 trapezes, $R_{hi}, i \in \{1,\ldots,M_z\text{-}1\}$ and the average hydraulic radius, R_h, are evaluated, respectively, with the relations:

(13) $$R_{hi} = A_i / P_i,\ i \in \{1,\ldots,M_Z - 1\},$$

In the hypothesis of neglecting the interaction between vertical planes (strands) with different speeds, the speed, W_i, and flow modules, K_i, related to the M_Z -1 trapezes are evaluated with the following relations:

(14) $K = A_i W_i = A_i C_i \sqrt{R_{hi}},\ i \in \{1, \ldots, M_z - 1\}$,
$W_i = C_i \pi \sqrt{R_{hi}}$

The coefficient of Chezy, C_i, is evaluated with a monomial formula of form:

(15) $$C_i = \frac{1}{\bar{n}_i} \left(R_{hi}\right)^{\chi_i},$$

in which $\bar{n}_i$, $\bar{n}_i = \left(n_i + n_{i+1}\right)/2$, represents the coefficient of roughness pertaining to the wet perimeter P_i; χ_i - the power exponent is after Manning 1/6, and after Pavlovski

(16) $\chi_i = 2{,}5\sqrt{\bar{n}_i} - 0{,}13 - 0{,}75\sqrt{R_{hi}}\left(\sqrt{\bar{n}_i} - 0{,}1\right)$

For natural, uneven roughness, an equivalent roughness coefficient is considered to be n_{ech}:

(17) $n_{ech} = e^{0{,}6667 \cdot \ln\left(P_n / P\right)}$ cu $P_n = \sum_{i=1}^{M_Z - 1} P_i \cdot e^{1{,}5 \cdot \ln \bar{n}_i}$

The hydraulic slope on the bed section delimited by the consecutive cross sections *j* and *j* + 1, $I_{h\,j,\,j+1}$, is evaluated with the relation:

(18) $I_{h\,j,j+1} = \left(Q_{j,j+1} / K_{j,j+1}\right)^2$, and

$Q_{j,j+1} = \left(Q_j + Q_{j+1}\right)/2$ şi $K_{j,j+1} = \left(K_j + K_{j+1}\right)/2$

The velocity module, averaged on the elementary (trapezoidal) sections, $\bar{V}_i$, is evaluated with the relations:

(19) $\bar{V}_i = \tilde{W}_i \cdot \sqrt{I_h}$, cu $i \in \{1, \ldots, M_Z - 1\}$

The average velocities in nodes (4), *V*i, were evaluated with the following weighted mean relationship:

(20) $V_i = \frac{A_{i-1} \cdot \bar{V}_{i-1} + A_i \cdot \bar{V}_i}{A_{i-1} + A_i}$, cu $i \in \left[\nu_S + 1, \nu_D - 1\right]$

Water flow in rivers is non-stationary and is mathematically modelled by the Saint Venant equations, represented by the continuity equation and the dynamic wave equation. At the initial moment, t1, the assumption of the constant and unevenly variable flow assumptions is accepted. For the deduction of the differential equation of the gradually varied motion, the system Saint Venant is considered $\partial Q/\partial t = 0$ and $\partial Z/\partial t = 0$, resulting in:

(21) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\alpha \frac{Q^2}{A}\right) + g\,A\left(\frac{\mathrm{d}Z}{\mathrm{d}x} + I_{\mathrm{h}}\right) - \left(u_q - \frac{Q}{A}\right) \cdot q = 0$,

and $q = \frac{\mathrm{d}Q}{\mathrm{d}x}$

Equation (21) must be solved with the following contour condition:

(22) $x = x^0; Z = Z^0$

where x^0 and Z^0 represent known values for the spatial coordinate x and respectively the level of the water level Z (for the downstream section, x0 = x0 = 0, and for the upstream section, $x^0 = x_0 + L$, with *L* the length of the river section for the area

For the numerical solving of the Cauchy problem defined by equations (21) and (22), equation (23) was developed in finite differences, with the step Δx following the x-x axis, inversely oriented to the flow direction. Thus, considering the hydraulic slope Ih expressed in a relation of the form (18), and for the area A and the Q flow the mean values on the length section Δx, in the case of the u_q neglect, between the hydraulic parameters in the upstream ends (index $_{am}$) and downstream (index $_{av}$) of the length section Δx can be determined the equation:

(23)

$$Z_{am} = Z_{av} + \frac{1}{2g}\left(\alpha_{av}\frac{Q_{av}^2}{A_{av}^2} - \alpha_{am}\frac{Q_{am}^2}{A_{am}^2}\right) + \frac{\left(Q_{av} + Q_{am}\right)^2}{\left(K_{am} + K_{av}\right)^2} \cdot \Delta x$$

The hydraulic - mathematical model and the numerical model, represented by the equations (22) and (23), formed the basis of the computer program MPGV_ River_Bed.m. The computerized software package implements the hydraulic-mathematical and numerical model in the MATLAB programming environment. It consists of the computer program MPGV_ River_Bed.m and seven user functions: 1 ° myfun_Albie.m, 2 ° myfun_Tet_Delt_BetR.m, 3 ° myfun_hU.m, 4 ° myfun_hCR.m, 5 ° myfun_Z_Am.m, 6 ° myfun_Tip_Remuu.m, and 7 ° myfun_Efort_Forfecare.m (Bălan A., 2016).

The user function *myfun_Efort_Forfecare.m* determines the shear stress of each of the nodes (*y*i, *z*i), depending on the velocity *V*i, the *h*i depth and the roughness coefficient *n*i.

The input data in *MPGV_ River_Bed.m* were: a - general data, *g*, ρ; b - longitudinal profile data, (*y*j, $z_{tv,\,j}$); c - cross-section data, (*y*i, *z*i) and *n*i; d - hydrological data, *Q* flow and water level share in the downstream section, *Z*av; e - data concerning the coefficients *a*1, *a*2 and *a*3 related to the functions θ = θ (βr) and δ = δ (βr) (Bălan A., 2016).

The output data from *MPGV_ River_Bed.m*, both tabulated and graphical, were systematized as follows: a - geometric and hydraulic parameters considered in MPGV (Table 1 - Columns 6 and 9 ÷ 15), *I*tv, *K*nec, *Z*u , *h*u, *B*cr, *A*cr, *Z*cr and *h*cr; b - geometric and hydraulic parameters determined in MPGV, *B*, *A*, *P*, *R*h, *C*, βr, θ, δ, *K*, *V*, *Z*GV, *h*GV; c - geometric and hydraulic parameters for the shear stress assessment at bottom y_i, z_i, h_i, V_i, τ_{bi}; d - longitudinal profiles through the real and theoretical bed of the river bed; e - longitudinal profiles representative in the progressive varied permanent movement; f - cross-sections through the bed, indicating the line of water gloss in MPGV; *g* - shear stress values (Bălan A., 2016).

The case study was drawn up on a section of the Moldova river with the length L = 287.36 m, which includes the sub-crossing area of the Timisesti-Iasi pipeline. The calculation data entered in the curriculum were: Q = 493 m^3/s water level share in the downstream section 255.29 mNMN, *n* = 0.04036.

The results obtained following the running of the program for a series of characteristic debits on the Moldovan River are presented in detail in (Bălan A., 2016). In *Table 1*, the data for the characteristic white matter parameters used to determine the type of curve line on each calculation section are shown in part.

Table 1

Geometric and hydraulic parameters of the bed in MPGV in the profiles P1 ... P3, Q = 493,0 m^3/s (Balan, 2016)

Ncr. *j*	*x* [m]	*B* [m]	*A* [m^2]	*P* [m]	*R$_h$* [m]	*C* [m^{0,5}s^{-1}]	*K* [m^{3}s^{-1}]	*V* [m s^{-1}]	*Z$_{GV}$* [m]	*h$_{GV}$* [m]
1	*2*	*4*	*5*	*6*	*7*	*8*	*12*	*13*	*14*	*15*
1	0,00	145,01	378,93	146,35	2,589	31,444	17774,35	1,301	255,29	4,11
2	35,83	132,46	360,50	135,41	2,662	31,504	16841,14	1,368	255,31	4,12
3	117,19	141,59	367,36	143,51	2,560	31,420	17262,74	1,342	255,41	4,20

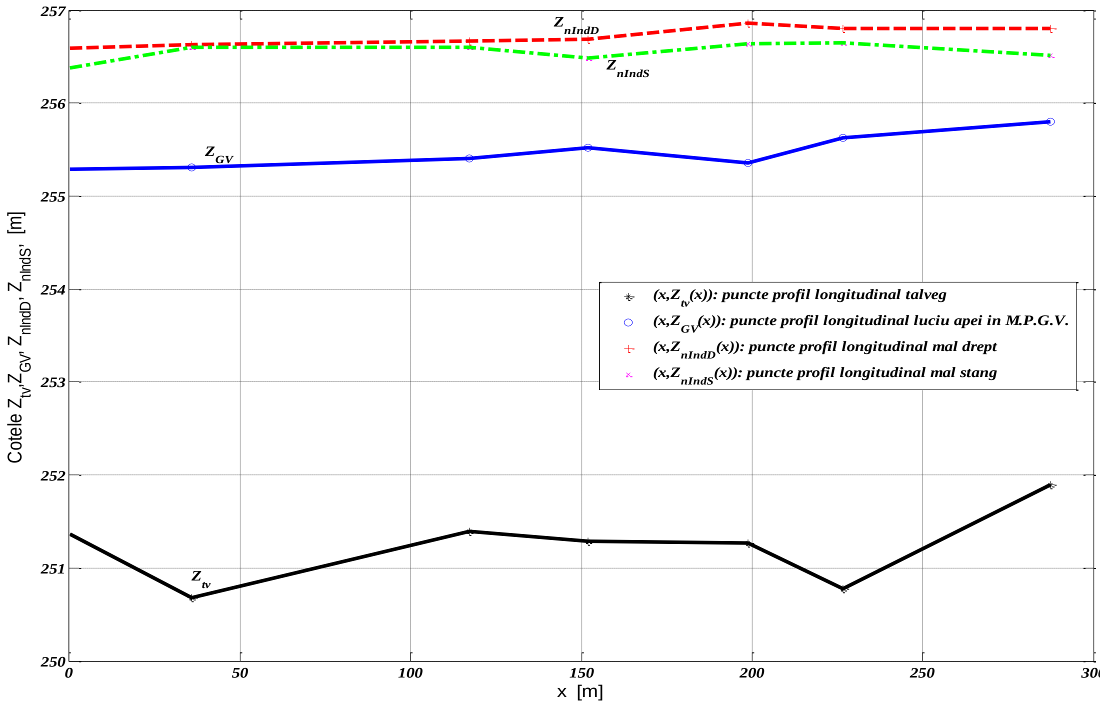

Figure 4 **The longitudinal profile calculated by indicating the bottom line and river banks (Z$_{tv}$ (x), Z$_{nindS}$(x), Z$_{nindD}$(x)) and of the water gloss in MPGV (Z$_{GV}$(x)) for Q = 493 m^3/s (Balan A. 2017)**

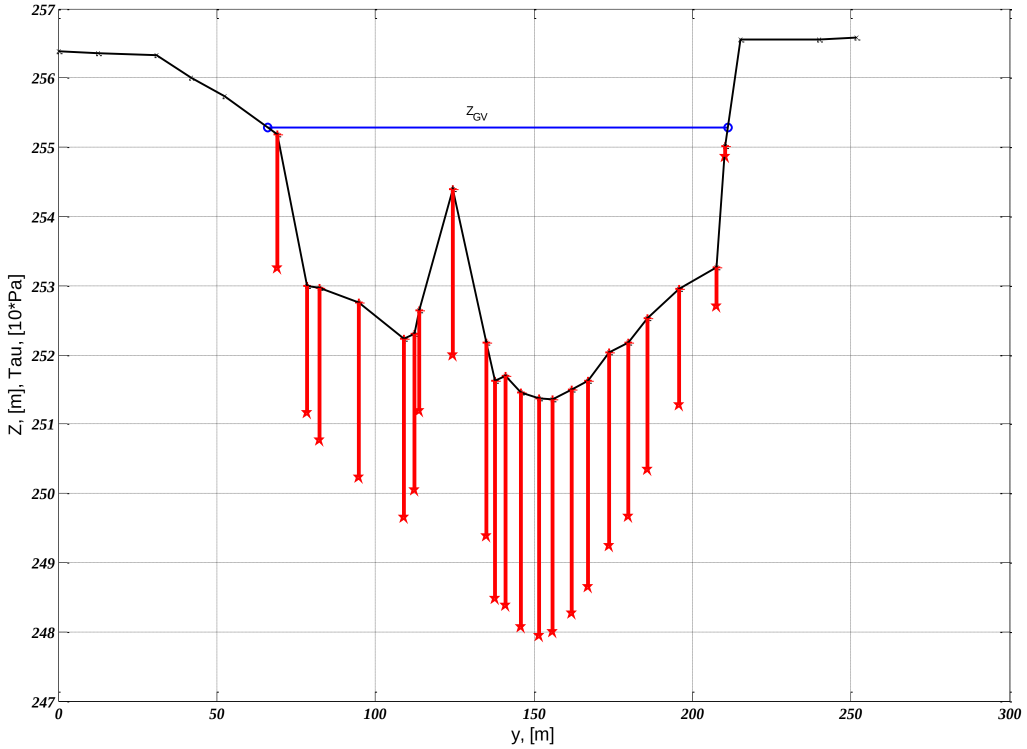

Figure 5 **Pc1 transverse profile indicating the line of water gloss in MPGV and the shear stress size**

The unitary shear effort on the perimeter of the bed obtained with the elaborated hydraulic-mathematical model aims at comparing them with the admissible limit values in order to assess the erosion depths (*table 2*) (Cioc D., 1981, Mitoiu C., Marin G., 1999, Hancu S., 1981).

Table 2

Bottom shearing effort on Pc1 profile (P7), nodes 1 ... 10, for Q = 493.0 m^3/s, Z_{GV} = 255.29m and I_h = 0.00768

(Balan A. 2017)

No. *cr.*	y_i [m]	z_i [m]	h_i [m]	n_i [$s \cdot m^{1/3}$]	V_i [m/s]	τ_{bi} [Pa]
1	*2*	*3*	*4*	*5*	*6*	*7*
1	69,170	255,190	0,098	0,0403	0,7499	19,4081
2	78,580	252,990	2,298	0,0403	1,2332	18,3604
3	82,510	252,970	2,318	0,0403	1,3533	22,0479
4	94,800	252,760	2,528	0,0403	1,4702	25,2779
5	108,930	252,240	3,048	0,0403	1,5365	25,9395
6	112,200	252,300	2,988	0,0403	1,4272	22,5302
7	113,950	252,640	2,648	0,0403	1,1244	14,5600
8	124,550	254,400	0,888	0,0403	1,2050	24,0646
9	135,010	252,180	3,108	0,0403	1,6032	28,0596
10	137,610	251,630	3,658	0,0403	1,7479	31,5882

The elaborated hydraulic-mathematical model assesses the conditions for the occurrence of the hydrodynamic erosion phenomenon in the riverbeds on the basis of topographical, hydrological, hydraulic and geotechnical surveys. The model is applicable in various conditions of morphological transformation of riverbeds.

Figure 6 **Upstream view of the study area on the Moldova River with erosion zones in the riverbed and bank (**photo: Luca M., year 2014).

The data obtained can be used in design calculations of hydro-technical works in riverbeds, shore defence works, bridges, etc.

CONCLUSIONS

1. The phenomenon of erosion of riverbeds is strongly influenced by the variation of the hydrological regime in the river basin of the Moldova River.

2. The elaborated hydraulic-mathematical model aims at determining the unitary shear forces on the perimeter of the bed and its comparison with the admissible limit values in order to assess the erosion depths.

3. The hydraulic-mathematical model for assessing the conditions of hydrodynamic erosion phenomena in river beds implies carrying out in-depth topographic, hydrological and geotechnical studies in order to obtain viable design data.

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